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SUMMARY

This review paper is the consequence of plasma introduction as a medium wuth specific electromagnetic properties in the problems of diffraction. It is done with the view of the newly opened up areas of this field, but clearly concentrating on the diversified requirements for practical applications, and, for space scientists, for radiocommunication in the presence of ionization irregularities, and, more particularly, with regard to the ionosphere.

It contain four major sections, each divided into sub-sections, and covering all aspe s of plasma boundaries, cavities and states of homogeneities or inhomogeneities. It goes as follows:

INTRODUCTION

1. LIMITING CASES OF LOW AND HIGH PLASMA DENSITY
   - Born Approximation
     Dense Plasma with Sharp or Weakly-Blurred Boundaries. Impedance Description

2. NUMERICAL COMPUTATIONS AND SHORTWAVE APPROXIMATION FOR A PLASMA OF ANY DENSITY

   - Reflection from a Uniform Nonabsorbing Sphere
   - Weak Plasma Boundary Washing
   - Plasma with Strongly Washed Boundary

3. RESONANCE SCATTERING ON SMALL PLASMA OBJECTS

   - Dipole Resonance in Cold Plasma

4. DIFFRACTION ON BODIES PLACED IN A NONUNIFORM PLASMA

   This section includes some comments with regards to the ionosphere.

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INTRODUCTION

General Remarks. The "introduction" of plasma as a medium with specific electromagnetic properties* in the problem of diffraction began relatively recently, no more than a couple of decades ago. Nevertheless, inasmuch as it opened up a new and broad area for the efforts of theoreticians, and also in connection with the diversified requirements of practical applications, such as the microwave diagnosis of laboratory plasma, the radiation acceleration of plasma clusters, the location and radiocommunication in the presence of ionization irregularities in the atmosphere and so forth, the number of publications in this field has exceeded a long time ago the bounds of the easily rewieable and it continuous to grow rapidly.

In the present brief review no detailed and exhaustive aim is laid down for the description of the contemporary state of the problem as a whole. The circle of the questions considered encompasses a certain, to a certain degree substantiated and independent part of them related to the regular and isotropic stationary plasma (i.e., not fluctuating and not magnetoactive) with a purely electron polarizability, assumed, as a rule, to be independent of the magnitude and structure of the field and described by the complex dielectric constant**

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2 - i\omega\nu},$$

where ω is the field frequency (the temporal factor $\exp(i\omega t)$ being assumed), ν is the effective collision frequency, $\omega_p = (4\pi e^2 N/m)^{1/2}$ is the plasma frequency (the Langmuir frequency), e is the charge, m is the mass of the electron and N is their concentration.

Relationship with Classical Problems of Diffraction. In a such a narrow statement of the problem the diffraction on plasma appears in essence as a certain generalization of classical diffraction problems to dielectric bodies (inhomogenous in the general case) with $\mathrm{Re}\,\varepsilon < 1$, so that all exhaustive analytical solutions could be carried nearly automatically to plasma objects. This concerns quite a few exact solutions for bodies of simplest shape (plane layer, sphere, cylinder [2]), as well as solutions obtained by approximate methods (shortwave and longwave approximations, Born approximation etc.). In case of weak field penetration into plasma, that is, for sufficiently great negative values of ε, it is found to be possible to base ourselves on simpler problems of diffraction on bodies with impedance boundary conditions.

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* A sufficiently detailed enumeration of plasma medium's singularities may be found, for example, in the Ginzburg's book [1].

** An important exception will be constituted, however, by so e questions of resonance interaction, for which the nonlinear effects and the spatial dispersion, conditioned by thermal motion, may have a principle significance (see Section 3).

It is indispensable, however, to bear in mind that the classical solutions, related as a rule to the case $\varepsilon > 1$, must, generally speaking, transfer into the region $-\infty < \varepsilon < 1$ with a well known caution, and in a series of cases even after appropriate reexamination. Two basic factors may be outlined, which are capable to hinder the direct transfer of the results from one region of values of ε to another. Firstly, this is a character variation of the spectrum of localized and quasilocalized solution of superficial wave type. In particular, for $0 < \varepsilon < 1$ the wavelength in the plasma is found to be greater than in the surrounding medium (vacuum) and, consequently, to earlier localized (for $\varepsilon > 1$) fields now correspond emitting, or the so called waves with leakage. As one passes to negative values of ε, there again takes place a qualitative variation in the character of localized solutions: "true" superficial waves appear, which are compressed to plasma boundary, return waves, waves with complex propagation constants. Obviously, all this affects substantially the formation of a diffraction field of the objects with great extension and compels us to construct for them anew a system of proper modes. In the course of the latest years such a work was performed relative to simplest plasma systems (plane-parallel plates [3-5], cylinders [6], and, to a known degree, a qualitative clarity was introduced into this question, without which there could hardly be any possibility of interpreting the structures of fields in complex systems.

The second factor is constituted by the singularities of field behavior in the neighborhood of plasma resonance surfaces ($\varepsilon = 0$), which for different frequencies may be disposed practically in any portions of the plasma object. As follows from the consideration of simplest plane-laminar models [7], the structure of plasma-resonance region, i. e. the character of ε transition through zero, determined its shielding action and the magnitude of resonance losses (which in case of linear transition are inversely proportional to its curvature [8]). On the other hand, the spatial dispersion [9, 10], manifest by the intense excitation of longitudinal plasma waves becomes substantial earlier of all precisely on these "zero" surfaces, where the component of the electric field E, parallel to $\nabla\varepsilon$, has a sharp maximum, and the nonlinear effects, linked with the deformation of plasma equilibrium distribution under the action of the field does likewise [11].

In order to be able to better concentrate on the specifically "plasma" problem (which obviously is by no means exhausted by the two noted factors), we shall consider the diffraction problems themselves in a simplest possible setup, digressing, in particular, from the singularities in the diffraction of localized [12 - 14] and pulsed fields [15], and from diffraction on combined systems (metal surrounded by plasma) [16]. Left aside are also the numerous problems of traditional classical diffraction, such as the distribution of illumination near the region of shadow boundary behind the scattering object, the exit from the Fraunhofer zone, the diffraction on body surface breaks and so forth. We shall dwell only upon scattering of plane or quasi-plane monochromatic waves on simplest structures (mostly with spherical or cylindrical symmetry) and on simplest diffraction characteristics of objects, i. e. the effective scattering cross-sections (mainly backscatter).

In view of the diversity of parameters characterizing the properties of the scattering plasma object (shape, dimensions, maximum value and character of density distribution, etc.), it would be extremely difficult to sustain a rigorous classification of problems over the extent of the entire review without indulging in a useless formalism and avoiding unnecessary repetitions. Thus, although in the expose sequence, adopted by us, there is perceived a certain original classification principle it is interwoven by the magnitude of plasma density with a classification according to other criteria, say by the magnitude of the ratio of characteristic dimensions to the wavelength, by methods of solution and, finally, simply by the character of plasma participation in diffraction. The latter found its expression in a special subsection of section 4, where plasma is viewed not as an object of diffraction, but as an inhomogenous medium filling a significant part of the course between the emitter and the scattering object. Here the principal attention is given not to the properly diffractional aspect of the problem, but to the transforming properties of the plasma medium where diffraction takes place.

1. LIMIT CASES OF LOW AND HIGH PLASMA DENSITY

Born Approximation. Simplest of all for the investigation is the case of rarefied plasma $\omega_p \ll \omega$, sufficiently well described with the help of the first, so called Born approximation method of perturbations [17]. In this approximation (which, incidently, is equally valid for the solution of problems of scattering on weak ionization disturbances in a boundless uniform plasma [18]) the computation of scattered field amounts to search for Fourier transformations of dielectric constant perturbations, $\Delta\varepsilon(r)$. In particular, the differential cross-section σ, equal to the ratio of energy flux scattered per unit of solid angle, to the density of energy flux in incident wave, is determined by the expression

$$\sigma = \frac{k_0^4 \sin^2\psi}{16\pi^2\varepsilon_e} \left| \int\limits_V \Delta\varepsilon \exp\left[-i(\mathbf{k}-\mathbf{k}')\,\mathbf{r}\right] dv \right|^2 \quad (\Delta\varepsilon \ll \varepsilon_e), \tag{1}$$

where $\vec{k}$ and $\vec{k}'$ are respectively the wave vectors of the incident and scattered waves (equal in absolute values $k = k_0\sqrt{\varepsilon_e}$, $k_0 = \omega/c$), ψ is the angle between the direction of scattering ($\vec{k}'$) and the electric vector of the incident wave, $\Delta\varepsilon = \varepsilon - \varepsilon_e$, ε_e is the dielectric constant of the surrounding medium.

By comparison with standard media (non-dispersing), the plasma singularity consists here in a different frequency dependence of scattering intensity and a greater variety of distributions $\Delta\varepsilon(\mathbf{r})$ offering practical interest. Let us note that the various concrete distributions, mainly for objects of quasi-cylindrical shape, are considered in the works [8, 18-20].

As is clearly seen from (1), the magnitude and the angular dependence of σ are determined essentially by the relation between the wavelength in the medium and the characteristic dimension of the scattering region $\underline{a}$. For greater wavelengths ($ka \ll 1$) scattering has a purely dipole character, and its intensity at $\varepsilon_e = 1$ is proportional to the square of the total number of electrons in the object and contrary, say, to the rayleigh case ($\sigma \sim \omega^4$), for $\omega >> \nu$ it generally does not depend on frequency (obviously within the limits $\omega >> \omega_p$).

In the opposite limit case ($ka >> 1$) nearly all the scattered power is concentrated in a narrow cone of angles near the direction $\vec{k}' = \vec{k}$. In the reverse and lateral directions scattering is quite weak and is mainly determined by reflection from those regions where concentration N or its derivatives vary noticeably over the wavelength. Let us illustrate this by an example of symmetrical distribution $N(r)$ having only 1 singular point $r = a$, in which there exists a finite number $(m - 1)$ of continuous derivatives, while the derivative of the m-th order has a jump $\Delta (N^{(m)})$. Effecting in (1) a consecutive integration by parts, we shall obtain in the shortwave approximation $\left(ka \sin \frac{\theta}{2} \gg 1\right)$

$$\sigma = 8\pi^2 r_0^2 a^2 [\Delta(N^{(m)})]^2 \sin^2 \psi \frac{1 + (-1)^m \cos\left(4ka \sin \frac{\theta}{2}\right)}{\varepsilon_e \left(2k \sin \frac{\theta}{2}\right)^{2m+4}} \tag{2}$$

($m = 0, 1, 2, ..$), θ is the angle between vectors $\vec{k}$ and $\vec{k}'$, $r_0 = e^2/mc^2$ is the classical radius of the electron. For a smooth concentration variation (in the wavelength scale) and all its derivatives, the backscatter cross-section is found to be exponentially small ($\sim e^{-(ka)^2}$).

It should be taken into account that in the shortwave region the first Born approximation may result insufficient on account of error accumulation in the wave phase, passing through the object. This is why the region of applicability of formula (2), for example, is bounded by the inequality

$$\int_0^a k_0 \Delta\varepsilon dr \ll 1$$

in the case of disruption of which the disposition of lobes of scattered power's diagram is incorrectly defined by this formula. However if we take interest only by the average intensity over a single lobe, the condition $\Delta\varepsilon << \varepsilon_e$ may as previously by considered as a sufficient criterion of precision.

Dense Plasma with Sharp or Weakly-blurred Boundaries. Impedance Description. The first, quite obvious and the most utilized approximation for dense plasma ($|\varepsilon| >> 1$) with sharp boundaries, in approximate estimates consists in its substitution by an ideally conducting medium. As a rule, it allows us to estimate the scattering cross-section with a sufficient degree of precision. It must, however, be borne in mind that by ascribing the plasma surface a zero impedance, we ignore, by the same token, its capability of directing the surface waves, which may, in a rather broad wavelength range, provide a substantial error when computing σ even for comparatively high ratios (we shall further dwell upon this subject in the following section).

Naturally, more precise results are provided by setting on plasma surface general boundary conditions of impedance type. Inasmuch as the diffraction problem itself becomes then more or less standard, we shall limit ourselves here to enumeration of different variants of such conditions.

For a small "skin-layer" thickness $\delta = c/\omega\sqrt{|\varepsilon|}$ (by comparison with the characteristic dimensions of the object $\underline{a}$) the values of surface impedance Z are found from the solutions of the corresponding plane problems, and, for a plasma of moderate density, they are found to be dependent on the structure and polarization of the external field (impedance with spatial dispersion). As to the case $|\varepsilon| \gg 1$, we may assume for Z the universal expression $Z = \varepsilon^{-1/2}$. Hence, in particular, for $\omega_p \gg \omega \gg \nu$ we have $Z = (\omega/\omega_p)(i + \nu/2\omega)$.

The plasma boundary washing, taking place in real conditions, always results in a certain modification of impedance boundary conditions. Thus, for a uniform object with $\mathrm{Re}\,\varepsilon = \varepsilon_0 \ll -1$, surrounded by a thin transition layer of thickness $l \ll \delta \ll a$ with linear variation of concentration along the transverse coordinate x ($\varepsilon = x/l_0 - i\nu/\omega$, $l_0 - l \leqslant x < l_0$, $1 - l/l_0 = \varepsilon_0$, it is possible to obtain the following general relation, linking the tangential (t) and normal (n) components of the external field at boundary (at $x = l_0$):

$$E_t = \left(\varepsilon_0^{-1/2} + ik_0 l_0\right)[nH_t] + I\nabla_t E_n, \tag{3}$$

where $I = \int_{l_0-l}^{l_0} \varepsilon^{-1}\,dx$. In a quasi-static case ($k_0 a \ll 1$) this relation passes into a combined condition for the scalar potential: $\varphi - I\dfrac{\partial\varphi}{\partial n} = \mathrm{const}$. It is material that the quantity I has a finite imaginary part $\mathrm{Im}\, I = \pi l_0$ (for $\nu \ll \omega$), independent of ν, so that even at formal threshold transition to $\nu = 0$, the energy losses inside the transitional layer (the so called "resonance" losses in the neighborhood of the point $\varepsilon = 0$), are in the general case not zero, as this follows from (3) (see Division 3).

The opposite limiting case $l \gg \delta$ is examined in the works [21, 22], in which the incoming half-space impedance with linearly rising concentration is computed for the cases of inclined TE- and TM-type incident waves. In particular, for $k_0 l_0 \gg 1$, we have with a precision to terms of higher order of smallness

$$Z = \begin{cases} iA\,(k_0 l_0)^{-1/3} & \text{(TE-waves)} \\ iA\,(k_0 l_0)^{-1/3} + \pi (k_0 l_0)^{-1}\sin^2\vartheta_0 & \text{(TM-waves)} \end{cases} \tag{4}$$

(ϑ_0 is the angle of incidence, $A = \Gamma(1/3)/3^{1/3}\,\Gamma(2/3)$).

For external fields with TM-polarization ($H_n = 0$, $E_n \neq 0$) the impedance boundary conditions may be also set on objects with zero permeability [5]:

$$Z = -i\infty \quad (H_t = 0 \text{ is an "ideal magnetic conductor"}) \tag{5}$$

and, in the presence in the transitional layer of a surface where ε has zero above the first order ($\varepsilon = (x/l_0)^n$, $n > 1$) [7],

.../..

$$Z = -i(n+1)(k_0 l_0)^{-1}. \tag{6}$$

2. NUMERICAL COMPUTATIONS AND SHORTWAVE APPROXIMATION FOR A PLASMA OF ANY DENSITY

The peculiarities of scattering on plasma with intermediate values of density are illustrated below on the basis of the results of certain numerical calculations and calculations performed in geometrooptical approximation (taking into account the refraction divergence and distortion of rays). We shall consider at the outset the plasma formation of simplest shape, the uniform sphere, one of the few objects for which the precise analytical solution is known for any ε, and then we shall discuss the influence of various complicating factors (weak or sharp washing of the boundary, disruption of spherical symmetry of density dsitribution).

Reflection from a Uniform Nonabosrbing Sphere. This was examined in [23] by way of numerical summation of series by spherical harmonics, in whose form appears the precise solution. Plotted in Fig.1 is the characteristic graph of the dependence of radar scattering cross-section $\sigma_{p\text{л}} = 4\pi\sigma(\theta=\pi)$ (normalized to the geometrooptical value $\sigma_0 = \pi a^2$ for a metallic sphere of same radius a) on $N/N_{cr} = \omega_p^2/\omega^2$ for a rather large ratio of radius to wavelength ($k_0 a = 40$). In the region $N \ll N_{cr}$ the value of $\sigma_{p\text{л}}$ rises monotonically with the concentration. As the concentration approaches the critical value N_{cr} $\sigma_{p\text{л}}$ begins to oscillate strongly (the drops reach 1 to 2 orders), though, as an average, it rises as previously with the increase of N. These oscillations are conditioned by the appearance of phase onrush (propagation phase increase) on sphere's diameter on account of concentration. At $N = N_{cr}$ ($\varepsilon = 0$) the radar cross-section coincides with σ_0 and remains invariable up to the value $N \simeq 4N_{cr}$, beginning with which the dependence $\sigma_{p\text{л}}(N)$ again becomes oscillating, deviating from σ_0 by about one order. As N increases further, the swing of oscillations gradually decreases and for $N \gtrsim 10^2 N_{cr}$ there no longer is any distinction between $\sigma_{p\text{л}}$ and σ_0. Analogous oscillations $\sigma_{p\text{л}}$ take place also at sphere radius variation, whereupon they remain significant through the values $k_0 a \sim 10^2$.

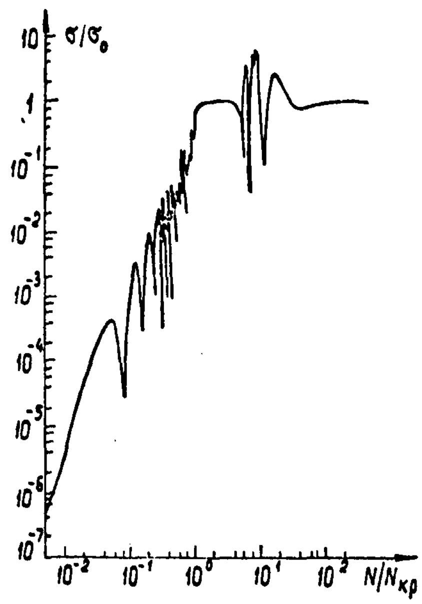

Fig.1

Anomalously large departures of the ratio $\sigma_{p\text{л}}/\sigma_0$ from the unity for $N > N_{cr}$ may be explained by resonance excitation of feebly emitting quasi-superficial waves directed by plasma boundary and the interference of their emission fields with the "regular" part of the scattered field. At plasma boundary surface waves may exist for $\varepsilon < -1$ ($N > 2N_{cr}$). The fact that on the graph presented oscillations begin from a doubled value of N is apparently linked with either a significant increase of interval N variation over the width of the corresponding resonance peaks, assumed at computations, or with the insufficiency of the number of series' terms taken into account (as $\varepsilon \to -1$, the width of peaks approaches zero and the required number of terms of series approaches the infinity*). Nevertheless, although the omission of high-order resonances in the vicinity of $\varepsilon = -1$ was indeed the consequence of a certain incorrectness of computations, it has a specific practical sense, inasmuch as even a very weak absorption in the plasma already leads to a strong damping of surface waves with a great deceleration (as $\varepsilon \to -1$ their phase velocity approaches zero) and consequently to a total suppression of corresponding resonances.

Weak Plasma Boundary Washing. This does not practically affect the part of the scattered field which is not connected with any sorts of resonances, but because of the presence of the above-mentioned terminal losses inside the transitional layer, it leads to the appearance of an additional surface wave damping [27], weakening by the same token the resonance effects linked with its excitation. The ratio of imaginary (h") to the true (h') part of the propagation constant of the surface wave traveling along the washed boundary, is equal with a precision to a factor of the order of unity to the ratio of thickness of the transitional (linear) layer l_0 to the wavelength 1/h' ($h''/h' \sim h' l_0$), whence it follows that the condition for a strongwave damping during one passage around the sphere is $(h')^2 \, a l_0 \gg 1$. Thus, even during a comparatively weak boundary washing, the resonances of surface waves, and alongside with them the oscillations of the scattering cross-sections relative to $\sigma_0 = \pi \vec{a}^2$, in the region $N > N_{cr}$, $\vec{k}_0 a \gg 1$, are already vanishing. This is corroborated also by the results of numerical integration of wave equations, performed for spherically and cylindrically symmetrical formations with various laws of concentration decrease by radius [28 - 31].

Plasma with Strongly Washed Boundary. Geometrooptical Approximation. In the case when the decrease of the concentration in the transparency region ($\varepsilon > 0$) takes place sufficiently smoothly in the wavelength scale, the problem may be fully resolved in geometric optics approximation and amounts to the calculation of ray refraction in a medium with a nonuniform index of refraction $n = \sqrt{\varepsilon}$ [32 - 39]. At the same time, for scattering cross sections of radially symmetrical objects one succeeds in obtaining sufficiently simple general expressions [32, 35, 36]. In particular, for an inhomogenous sphere without losses

$$\sigma_{p\,\pi} = \pi \left[\int\limits_a^\infty \frac{dr}{n(r)\, r^2} \right]^{-2} (n(a) = 0; \;\; 0 < n(r > a)) \qquad (7)$$

Hence it follows that the often used substitution of a nonuniform plasma sphere by a metallic sphere of radius a on which $\varepsilon = 0$ for approximate estimates, always yields overrated values of back scatter cross-section ($\sigma_{p\,\pi} < \sigma_0 = \pi a^2$). Thus, for $\varepsilon(r \geqslant a) = 1 - a^2/r^2$ $\sigma_{p\,\pi} = 4\sigma_0/\pi^2$ [36]. For objects with sufficiently thin transitional layer ($\varepsilon = (r-a)/l_0$, $a \leqslant r \leqslant a + l_0$, $l_0 \ll a$) the refraction effects yield only small corrections to cross-section:

$$\sigma_{p\,\pi} = \sigma_0 (1 - 2l_0/a).$$

Note also that to functions (r), having at r = a a zero of second or higher order ($\varepsilon \sim (r - a)^m$, $m > 2$), corresponds an infinite divergence of the normal radial tube, so that in the considered approximation $\sigma_{p\pi}$ becomes zero.

The disruption of radial symmetry of plasma distribution naturally results in the appearance of $\sigma_{p\pi}$ dependence on the direction of irradiation [37 - 39]. In particular, for an object of which the surfaces of equal concentration are spheres with a common tangent [39], the quantity $\sigma_{p\pi}$ may be either greater or smaller than σ_0 at various irradiation angles.

Alongside with the refractional distortion of rays, a great influence may be exerted on the magnitude of the power scattered by an extended non-uniform object by losses in the plasma. In the geometric optics approximation their accounting for $\nu \ll \omega$ is performed sufficiently elementarily - by the mere introduction of the exponential multiplier

$$\exp\left(-2\int_L k_0 \operatorname{Im}\sqrt{\varepsilon}\, dl\right),$$

characterizing the absorption along the respective radial course (L). It is obvious that significant losses may take place over a sufficiently stretched course, strongly underrating the scattering cross-section even for small $\operatorname{Im}\sqrt{\varepsilon}$.

3. RESONANCE SCATTERING ON SMALL PLASMA OBJECTS *

Dipole Resonance in "Cold" Plasma. Scattering on particles whose size is small by comparison with the wavelength may, as a rule, be computed in the dipole approximation, determining the electric and magnetic dipole moments induced by the field of the incident wave (for two-dimensional cylindircal objects, equivalent linear currents (electric and magnetic), yielding isotropic scattering, must moreover be taken into account).

One of the most important peculiarities of scattering on small plasma objects is the possibility of resonance increase of the scattering cross-section in the frequency of proper electro-dipole oscillations. Generally speaking, no less important role may be played also by high multipole resonances in the presence of only radiation energy losses [8, 24, 25] with whose appearance the dipole approximation for a scattered wave becomes invalid. (Indeed, as the object's dimensions increase, these resonances continuously pass into the above-mentioned (Div.2) surface wave resonances. In essence either of them have the same quasi-static nature and they are conditioned in the final count by the presence in the plasma of elastic restoring forces of Coulomb origin). However, as the order of multipole oscillation in objects increases, when the latter are small, the relative share of inner losses such as particle collisions, collisionless damping) rises rapidly, which results in the suppression of higher-order resonances. We shall limit ourselves here to the consideration of electro-dipole-type scattering, of which the differential (σ) and total (σ_t) cross-sections are determined by the well known expressions

$$\sigma = |\alpha|^2 k_0^4 \sin^2\psi, \qquad \sigma_t = \frac{8\pi}{3}|\alpha|^2 k_0^4 \tag{8}$$

* (see Appendix)

(we have in mind the three-dimensional case; ψ is the angle formed by the direction of scattering with the dipole moment vector $\vec{P} = \alpha\vec{E}_0$, α is the polarizability factor, $\vec{E}_0$ is the amplitude of the electric field of the incident wave.

As was noted first in the Tonks' work [41], transverse "electrostatic" oscillations, capable of being excited by external fields, are also possible in a "cold" bound plasma, alongside with purely internal longitudinal oscillations in the plasma frequency ω_p. Their natural frequencies ω_0 always lie in the region $\omega < \omega_p$ (which already follows from the general condition of existence of proper statistical solutions $\int \varepsilon E^2 dv = 0$), and depend on their geometrical shape). Thus, for the dipole moment of a uniform sphere of radius a $\left(k_0 a \ll 1,\ k_0 \sqrt{|\varepsilon|}\, a \ll 1\right)$ in case of a real ε, we have directly from its quasi-static expression

$$P = \frac{\varepsilon - 1}{\varepsilon + 2} a^3 E_0 \tag{9}$$

the following values:

$$\varepsilon(\omega_0) = -2, \qquad \omega_0 = \omega_p/\sqrt{3}\,. \tag{10}$$

Analogously, for an infinite cylinder of radius a in a transverse field, we have

$$P_{\text{ел ял}} = \frac{1}{2}\frac{\varepsilon - 1}{\varepsilon + 1} a^2 E_0, \quad \varepsilon(\omega_0) = -1, \quad \omega_0 = \omega_p/\sqrt{2}\,. \tag{11}$$

It is obvious that the resonance values of dipole moments are not infinite. For any accounting of losses there appear in the denominators of the expressions brought out imaginary corrections defining the value of $P(\omega_0)$ and the width of the resonance line γ. In the absence of internal dissipation these corrections have a purely radiational origin and are easily determined by addition to the external field $\vec{E}_0$ of the proper field $\vec{E}_r$ of the radiation deceleration. In the three-dimensional case [42], p.245, we have

$$\begin{gathered} E_r = \frac{2}{3c^3}\dddot{P} = -i\frac{2}{3}k_0^3 P, \\ P = \alpha(E_0 + E_r) = \frac{\alpha E_0}{1 + 2i\alpha k_0^3/3}, \\ P(\omega_0) = -i\frac{3E_0}{2k_0^3}, \qquad \sigma_t(\omega_0) = \frac{6\pi}{k_0^2}. \end{gathered} \tag{12}$$

Therefore, the resonance values of P and σ_t, determined by losses on the emission, are found to be independent of the structure and dimensions of the scattering object. The radiation width of the resonance line γ_r is determined by the ratio of its characteristic dimensions to wavelength; for a sphere

$$\gamma_r = \frac{1}{3} \omega_0 (k_0 a)^3. \tag{13}$$

The influence of internal losses leading to additional widening of the line and to lowering of the resonance cross-section, depends essentially on the character of distribution of plasma concentration, or, to be more precise, to the degree of its boundary washing. As to losses on collisions between particles, their accounting is performed elementarily ($\mathrm{Im}\,\varepsilon \neq 0$) and yields an additional widening of the resonance line of homogenous objects with sharp boundary $\gamma_s = \nu$ (case of washed boundary being considered later).

More complex is the question of the role of thermal flow of electrons and of spatial dispersion linked with it. The corresponding analysis was conducted for opposite limiting cases of weak and strong plasma boundary washing in the scale of Debye radius.

Taking into account of motion in a plasma with sharp boundary leads first of all to the appearance of a link between the above-mentioned external fields with longitudinal (plasma) oscillations and allows us to describe the resonance effects caused by their excitation. These additional "plasma-wave" resonances were investigated for a uniform sphere and cylinder in hydrodynamic approximation (i. e. in case of weak spatial dispersion) on the basis of quasi-static description [43], as well as by way of corresponding generalization of well known plane wave diffraction on bodies made of standard dielectric [44 - 46]. The solution was sought by standard methods of variable separation, while the accounting of the spatial dispersion was reduced by the fact, that in a plasma there was introduced alongside with the transverse field a longitudinal one satisfying the independent wave equation. The relationship between the two fields was materialized by an additional boundary condition, i. e. the continuity of the normal component of the electric field. From the expressions thus found for the amplitudes of scattered waves it follows that the thermal motion does not practically affect the frequencies of electrostatic resonances and leads to the appearance of a series of additional resonance lines in the region $0 < \varepsilon \ll 1$. For dipole scattering on a sphere, the position of these lines is defined by the condition

$$j_1(k_p a) = k_p a j_1'(k_p a), \tag{14}$$

where j_1 is a Bessel spherical function, $k_p = \omega\sqrt{\varepsilon}/\sqrt{3}\,v_T$ is the wave number for the longitudinal field, v_T is the mean thermal velocity of electrons. An analogous relation for a cylinder is obtained from here by substitution of the spherical function by a cylindrical one.

The second important effect resulting in the thermal flow is the collisionless dissipation, essentially conditioned by electron collisions with plasma boundaries. This boundary dissipation may be investigated in the hydrodynamic approximation. The damping constants γ_b determined by it may be estimated qualitatively by way of kinetic consideration of oscillation of a plane plasma layer, partially filling the space between the planes of a plane condensor (resonances of both forms may also be possible in such a one-dimensional system [43]). In case of specular reflection of electrons from the boundary, for electrostatic and plasma-wave resonances γ_b is proportional respectively to the first and fifth powers of the characteristic frequency of electrons' collisions, with boundary v_T/a.

The washing of plasma boundary results in the increase of internal losses and, consequently, in the decrease of the effectiveness of resonance interaction. If the thickness of the transitional boundary region l_0 of the plasma exceeds significantly the Debye radius ($l_0 \gg v_T/\omega$), the longitudinal waves excited in it must satisfy their own kind of condition for emission, owing to strong Landau attenuation at periphery; this condition makes the occurrence of plasma-wave resonances impossible in objects with smooth and monotonic decrease of density from center to periphery. For spherically or cylindrically symmetrical distributions of plasma these resonances are absent even in the case when the seepage of longitudinal waves into peripheral regions with strong damping is made difficult by the presence of jumps of derivative concentration [10].

In regions with washed out boundary the electrostatic resonances, though not fully suppressed, also are found to be attenuated by comparison with the case of sharp boundary. The decrease of their divisibility is conditioned by great energy losses in the neighborhood of the surface $\varepsilon(\vec{r}) = 0$, where the electric field has a singularity of the type $1/\varepsilon$, lifted either by collisions of particles (for $\nu/\omega \gg (v_T/\omega l_0)^{2/3}$), or by spatial dis ion (at fulfilment of inverse inequality) [9]. In the first case [8, 24], the main dissipation mechanism is in the collisions, and in the second case [10] - in the energy transformation into longitudinal waves traveling toward the periphery. The total magnitude of losses inside the boundary layer and the widening of the resonance line γ_l conditioned by these losses are identical in both cases (for small ν and v_T); they are fully defined by the steepness of plasma density differential in the vicinity of the singular point. In particular, for a sphere, whose permeability is constant ($\varepsilon = \varepsilon_0$) for $r \leqslant a$ and increases linearly to the unity with derivative $d\varepsilon/dr = l_0^{-1}$ in the transitional layer. when $l_0 \ll a$, $\varepsilon_0 \sim -1$ we have

$$P = \frac{\varepsilon_0 - 1 - i\beta}{\varepsilon_0 + 2 + i2\beta}, \quad \beta = \pi\varepsilon_0 l_0/a, \quad \gamma_l = -\frac{1}{3}\omega\beta = \frac{2\pi}{3}\omega\frac{l_0}{a}. \tag{15}$$

Analogous expressions are obtained for a cylinder.

If the characteristic dimension of the boundary region is comparable with the dimensions of the object itself, the divisibility ω/γ_l becomes a quantity of the order of the unity and the amplitudes of scattered field increase at resonance by no more than 2 - 3 times.

These conclusions are in agreement with the data of experiments on the study of resonance scattering on meteor wakes: the oscillograms of the reflected signals show, as a rule, the presence of only one weakly expressed resonance [47].

When combining the smooth density decrease in the direction toward the peripheral region with its jump-like drop to zero at a certain boundary surface $\vec{r} = \vec{r}_b$ (which, for example, takes place in gas-discharge tubes), the region of damping of plasma waves, and alongside with it the foundations for the superimposition upon them the emission conditions disappear for $\varepsilon(\vec{r}_b) \ll 1$. Then the resonance effects linked with the excitation of standing plasma waves become again possible; their transparency region is now disposed at the periphery and is bounded by surfaces $\varepsilon(\vec{r}) = 0$ and $\vec{r} = \vec{r}_b$. This is precisely the way the so called Tonks-Dattner resonances must be interpreted [41, 48]. They were experimentally observed at interaction of high frequency fields with gas-discharge plasma. The theoretical computation of their spectrum conducted on adequate models yields quite satisfactory results relative to experiment from the standpoint of the general character of line disposition, as well as of that for the magnitude of intervals between them [49 - 52].

The nonlinear effects in condition of resonance interaction already become noticeable at comparatively small amplitudes of the external field and may be subdivided into two fundamental groups: 1) the appearance of higher harmonics of induced polarization and scattered signal [53] and 2) the so-called self-action processes expressed in the deformation of the stationary distribution of plasma parameters, defining the scattering characteristics in the basic frequency [11, 54].

One of the most important effects of the second group, capable of influence substantially the resonance properties of objects with washed boundary, is the deformation of spatial distribution of density near the plasma resonance point $\mathrm{Re}\,\varepsilon = 0$. On the one hand, the presence of a sharp maximum of averaged forces' potential $\Phi = 4\pi e^2 |\vec{E}|^2 / m\omega^2$ [55], and the high sensitivity of resonance losses to the magnitude of density gradient in that region on the other, render the resonance divisibility quite critical relative to field amplitude. The corresponding computations were conducted for the simplest model with plane-stratified density distribution $N(\chi)$, subject to Boltzmann law and characterized by local complex permeability $\varepsilon(\chi, |E(\chi)|^2)$ [11]. Analysis of the solution of the nonlinear (cubic) equation, determining the permeability distribution in the presence of the field, shows that for amplitude values of the external field $E_x^{(0)}$, exceeding a certain critical value

$$E_{\text{кр}} = \left(\frac{12{,}3 m^2 v_T^2 \nu^3}{e^2 \omega} \right)^{1/2}, \tag{16}$$

this distribution acquires in the region $\omega_p \simeq \omega$ the shape of a step, whose disposition relative relative to the level $\mathrm{Re}\,\varepsilon = 0$ depends on the amplitude in a hysteretic fashion: as it accrues, the transition through the point

$\mathrm{Re}\varepsilon = 0$ takes place by a jump; during the reverse amplitude decrease, the jump shifts into the region $\mathrm{Re}\varepsilon < 0$ and there appears a broad "plateau" with $\mathrm{Re}\varepsilon \sim \mathrm{Im}\varepsilon$. The value of electrostatic resonance divisibility respectively increases or decreases by comparison with the linear case.

"Electromagnetic'Resonances in Plasma Cavity. Up until now we did not mention the standard electromagnetic resonances capable of appearing at specific correlations between the wavelength inside the object and its dimensions. For plasma objects disposed in a vacuum, these correlations may be fulfilled only outside the quasistatistical region (for $\vec{k}_0 a \gtrsim 1$), where the oscillations divisibility is found to be quite low ($\gamma_r \sim \omega$) as a consequence of great losses. However, the situation changes substantially for an object constituting in itself rarefaction (cavity) of plasma. If in this cavity $\varepsilon \simeq 1$, while in the surrounding unperturbed plasma $0 < \varepsilon_e << 1$, the external wavelength results to be of much greater dimension of the cavity even upon fulfilment of the resonance condition, and, consequently, the losses to emission become quite small while the divisibility increases strongly. At the same time, contrary to plasma objects in a vacuum, the cavity boundary washing does not result in the appearance of additional internal losses and in the weakening of resonance.

The resonance frequencies of quasi-spherical and quasi-splindrical shapes of cavities lie, as a rule, in the region $\vec{k}_0 a \gtrsim 1$. The exception is only in the "zero" mode resonance considered in the works [56, 57] (yielding isotropic scattering) for a cylindrical cavity parallel to the electric field of the incident wave. For $\vec{k}_0 a << 1$, the condition of this resonance has the form $(\vec{k}_0 a)^2 \ln\left(1/\vec{k}_0 a\sqrt{\varepsilon_e}\right) = 2$.

4. DIFFRACTION ON BODIES PLACED IN A NONUNIFORM PLASMA

Discussed in this section are the problems to a known sense complementary to those considered earlier: emitter (1), scattering object (2) and emitter (3) are assumed to be disposed in an inhomogenous (nonuniform) plasma medium with permeability $\varepsilon(\vec{r})$; it is required to determine the transmission factor K_{123} characterizing the part of the received signal which is conditioned by scattering on the object. In the most general statement such kind of problems appear as being extremely complex. Here we shall limit ourselves to the consideration of some sufficiently typical particular cases referred to a medium whose properties vary slowly in the wavelength scale.

Field Transformation Factors. Apparent Scattering Cross-Section. A substantial simplification of the problem stated is obtained by the introduction of the following assumptions: 1) the scattering object and the receiver are disposed in the region of applicability of the geometric optics approximation respectively for the primary and scattered fields; 2) the emitted signal will hit the receiver along a single radial course (emitter-object-receiver); 3) the dimensions of the receiver, emitter and object are sufficiently small by comparison with the characteristic scale of medium's inhomogeneity, so that in the formation regions of their radiation patterns the medium is practically uniform (it should be possible to postulate without any limitation of generality that $\varepsilon_1 = \varepsilon_3 = 1$).

At fulfilment of these conditions the accounting of the inhomogenous medium amounts to the investigation of its transforming properties over corresponding radial trajectories, while the diffractional part proper of the problem looks the same as in a uniform medium. Effecting the required conversions of energy fluxes, and comparing their values in a uniform and inhomogenous media, we obtain the following expression for the transfer coefficient:

$$K_{123} = \frac{G_1 A_3 \sigma_{13}(\varepsilon_2)\left(T_{12}^{(E)} T_{23}^{(E)}\right)^2}{4\pi R_{12}^2 R_{23}^2}, \tag{17}$$

where G_1 and A_3 are the respectively the coefficient of directed action and the effective area of transmitting and receiving antennas; R_{12} and R_{23} are the distances from transmitter to receiver to the object; $T_{12}^{(E)}$ and $T_{23}^{(E)}$ are the amplitude transformation factors of electric field obviously linked with the so called focusing factors [58]. The first of them is equal to the ratio of electric field amplitudes induced by the emitter at the place of location of the object in the given inhomogenous medium and in the vacuum for identical R_{12} and fixed orientation of emitter's radiation pattern relative to the direction of exit of the ray hitting the object. The second coefficient is determined analogously for the field in the receiving zone with the difference, however, that here plasma surrounding the object ($\varepsilon = \varepsilon_2$) is taken for the hypothetical uniform medium and not the vacuum. The differential scattering cross-section σ_{13} (ε_2) is determined in the same uniform medium, whereupon directions 1 and 3 are chosen taking into account the refraction. In a number of cases the quantity σ_{13} (ε_2) is simply expressed by the cross-section σ_{13}^0 in the vacuum. In particular, for ideally conducting objects the similitude factor $f(\varepsilon_2) = \sigma_{13}(\varepsilon_2)/\sigma_{13}^0$ in geometrooptical ($ka \gg 1$) and quasistatic ($k_0 a \ll 1$) approximations is respectively equal to 1 and .

Therefore, in an inhomogenous medium the role of standard scattering cross-section is played by the quantity

$$\sigma_{13}^* = \sigma_{13}(\varepsilon_2)\left(T_{12}^{(E)} T_{23}^{(E)}\right)^2, \tag{18}$$

which may be called "apparent cross-section"; for a given σ_{13} (ε_2), its calculation amounts to the search for transformation factors of electric field amplitudes.

In the particular case of backscattering when irradiated by en elementary electric dipole, we have by virtue of reciprocity relations

$$T_{12}^{(E)} = T_{23}^{(E)} = T^{(E)}, \quad \sigma_{13}^* = T^{(E)4} \sigma_{11}(\varepsilon_2). \tag{19}$$

Rather detailed calculations of the coefficient $T^{(E)}$ in an illuminated region were conducted for linear and parabolic plane-stratified distributions $\varepsilon(z)$ [59]. One of the typical cases related to the linear layer ($\varepsilon = 1 - \alpha z$, $z > 0$), is shown in Fig.2, where the dependence $T^{(E)}(z)$ is plotted for rays emerging from the point $z = -h$ with different initial inclination angles

ϑ_0 to the axis z (dashed lines refer to rays having passed the caustic). Note here that parameter h = 0.725 was chosen so as to meet to some measure the conditions of the Earth's ionosphere. The curves are broken at the boundary of geometric optics applicability, where the quantity $T^{(E)}$ attains the maximum value (see below).

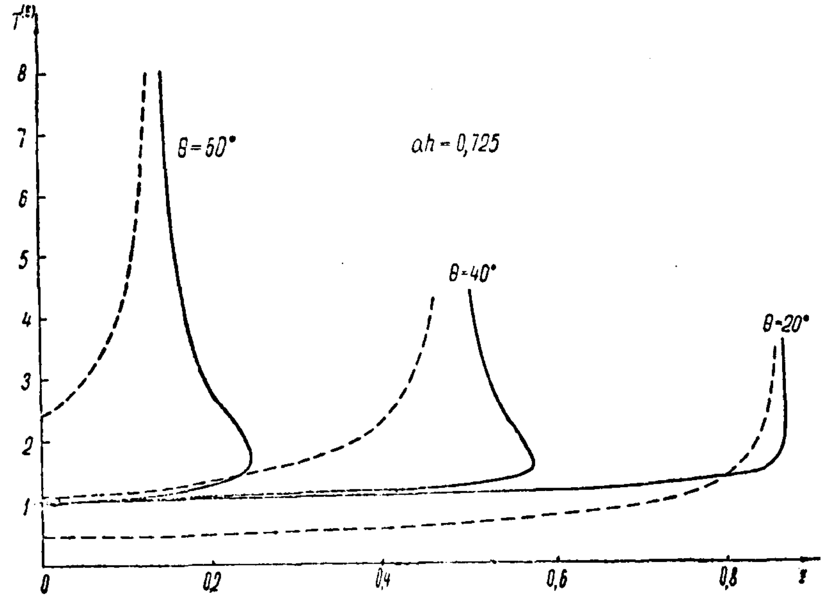

Fig.2

Point Scatterers in the Region of Geometric Optics Disruption. For objects small by comparison with the dimensions of field inhomogeneity, the method described remains valid (with some refinements) even in the case when they are disposed in regions where the geometrooptical approximation is inapplicable. Thus, in the case of electro-dipole-type scattering the expressions for the apparent cross section of backscattering σ_{11}^{*} and of the transmission factor K_{121} remain without change except for the fact that the quantity $\sigma_{11}(\varepsilon_2)$ is replaced by the product $\sigma_{11}^{(0)}f(\varepsilon_2)$, where $f(\varepsilon_2)$ is the ratio of object polarizability in media with $\varepsilon = \varepsilon_2$ and $\varepsilon = 1$ (the wave vector and the polarization of a plane wave relative to which σ_{11}^{0} is computed, are determined by the direction of the unperturbed electric field at the given point of the inhomogenous medium).

In the feebly inhomogenous plasma considered by us we may separate three characteristic regions in which geometric optics are inapplicable for a given position of the source: the near-caustic region, the shadow and plasma resonance regions (in the vicinity of $\varepsilon = 0$). The indicated regions differ among themselves by the structure of the field and the method of its calculation, though even here, as a rule, one may start from somewhat modified radial representations.

For a region near a caustic surface (the shape of caustic surfaces for various positions of the source and various forms of medium inhomogeneity was investigated in [60]) the corresponding method was expounded in [61, 62] (see also [63]). In essence, this method is that of standard Eyri functions, but their arguments and coefficients are computed by the standard formulas of geometric optics. It is necessary to stress that the region of geometro-optical approximation in fact extends through the first maximum of the Eyri function, where its exact values still remain close to asymptotic. Referring again to Fig.2 where curves $T^{(E)}(z)$ are plotted, let us remark that the maximum values of $T^{(E)}$ correspond precisely to the first maximum and are, as may be seen from the drawing, sufficiently great, so that at $\vartheta_{0} = 60°$ the gain in backscattering cross-section $\sigma^{*}_{11}/\sigma^{(0)}_{11}$ may constitute more than three orders. Note that the strong scatter effect near the caustic surface was first established in [64], applicably to fluctuations in the ionosphere.

Applicably to the region of caustic shadow the generalization of the geometric optics method is based upon the introduction of complex rays [65] (see also the review paper [66]). As to the resonance region ($\omega_p \simeq \omega$, $(\vec{E}\nabla\varepsilon) \neq 0$), the generalization of "complex" geometric optics is apparently possible for a sufficiently far withdrawal of that region from caustic surface in the case of linear transition of ε through zero and provided Whittaker functions are used for standard ones [67]. However, in the more interesting case of close disposition of caustic and resonance surfaces (when the resonance increase of amplitude becomes maximum [9]), one can not generalize the geometro-optical method by virtue of the absence of unified standard functions. Special consideration is also required of resonance regions with another character of transition through zero [7]. As this transition "smoothes out", i. e. as the number of derivatives becoming zero increases, the effect of resonance increase of the field gradually attenuates and vanishes completely in the limit case of a uniform layer with $\varepsilon = 0$. (At the same time if $\nu/\omega \to 0$ for all transitions excpet the linear, the surface $\varepsilon = 0$ is found be an ideal screen relative to TM-type fields). However, even in the case when this effect is expressed sufficiently strongly, the decrease of the similitude factor $\tilde{f}(\varepsilon_2)$ is hindering the significant increase of the apparent cross-section. Thus for a small metallic object ($\tilde{f}(\varepsilon_2) = \varepsilon_2^2$) for values of parameters close to ionospheric, the ratio $\sigma^{*}_{11}/\sigma^{(0)}_{11}$ approaches in the best case the unity, as $\mathrm{Re}\varepsilon^2 \to 0$.

One of the most curious example is the region of lowered concentration i. e. the plasma cavity considered in the preceding section as an object on which scattering may have a resonance character. If the wavelength in the surrounding plasma is small by comparison with the characteristic dimension of the cavity and with distances over which a substantial plasma density differential takes place inside it, all kinds of resonance events vanish completely and the problem of scattering is reduced to the study of ray refraction in the cavity itself and in the surrounding medium. The inhomogeneity of the latter then acquires an extremely important role, inasmuch as a direct inverse reflection from the cavity considered is absent in general in geometric optics approximation and the return of the scattered signal to the source is possible only in the presence by a distorted radial course in the corresponding manner. Applicably to scattering on rarefaction of electron density in the Earth's ionosphere, two characteristic courses of such form can be indicated: 1) as a result of refraction on cavity, the ray rolls out perpendicularly to ionosphere layers, reflects from the surface $\varepsilon = 0$, and returns to the source along the same trajectory; 2) at passage through the cavity the ray undergoes a strong rotation, which is stronger than in the first case, and drops on the reflecting layers of the ionosphere at an angle, ensuring its return to the source without secondary refraction on cavity.

When there is little difference in the indices of refraction of the cavity and of the surrounding medium, scattering in the inverse (return) direction still may be quite significant if the wave vector of the incident wave is oriented at the outset perpendicularly to the reflecting layers. At the same time, the backscattering cross-section may be found by way of direct calculation of distortions of the general phase front, determined without taking into account the distortion of rays [57].

**** T H E E N D ****

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[ADDENDA]

SOME INFRAPAGINAL ANNOTATIONS OMITTED FROM TEXT.

* (page 8) The formal condition for surface wave reonances in the vicinity of the point $\varepsilon = -1$ has the same form as for quasistatic multipole oscillations [24, 25]: $\varepsilon n + n + 1 = 0$; $n = 1, 2, 3, \ldots$ It is of interest that the scattering cross-section of a nonabsorbing sphere of any radius, the expression for which at resonance of n-th order always contains the addend $2\pi\ (2n + 1)/k_0^2$ as $n \to \infty$, i. e. as $\varepsilon \to -1$, becomes infinite (for the case of sphere of large radius this fact was first noted in [26]); the indicated singularity is, in truth, fully lifted by the introduction of as small an absorption as deemed desirable.

(page 9) The contents of this and of subsequent sections were expounded in abbreviated form in the review [40].

** (page 17).. insert omission:

Scattering on Objects of Large Dimensions (ka >> 1). When in feebly inhomogenous medium, it may be invsetigated directly in the geometric optics approximation without the above breakdown into "diffraction" and "refraction" parts. Such a division loses all sense the more so when the dimensions of the object (or to be more precise, of that part of it by which scattering is determined in the given direction) become comparable with the dimension of the inhomogenous medium. Attention should be paid here to works [68 - 70), where the locality principle is refined and the geometric theory of diffraction developed for objects with impedance boundary conditions.

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